Continuity

Limiting Value = Functional Value

$L.H.L = R.H.L = F(c)$

$f(x)$ at c

DOWNLOAD GAMMA CAREER CLASSES AAP!

limit

L·H·L = R·H·L = ℓ (say)

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = \ell$$

limit exist

$$\lim_{x \to c} f(x) = \ell$$

DOWNLOAD GAMMA CAREER CLASSES AAP!

Continuity of a function at a fixed point A real function f is continuous at a point c if

$$\lim_{x \to c} f(x) = f(c)$$

$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$ $f(x)$ at $x = c$ Continuous.

When C is any point in the domain of function.

DOWNLOAD GAMMA CAREER CLASSES AAP!

L·H·L = R·H·L = F(c)

Note:-

a.) Left Continuous at a point

If $\lim_{x \to c^-} F(x) = F(c)$ L.H.S = F^n $Value$

b.) Right Continuous at a point

If $\lim_{x \to c^+} F(x) = F(c)$ R.H.S = F^n $Value$

c.) Continuous at a point

F^n is left as well as right continuous.

d.) Discontinuous at a point

If Function is not Continuous.

e) Removable and Non- removable discontinuous

L·H·L = R·H·L ≠ F(x)

DOWNLOAD GAMMA CAREER CLASSES AAP!

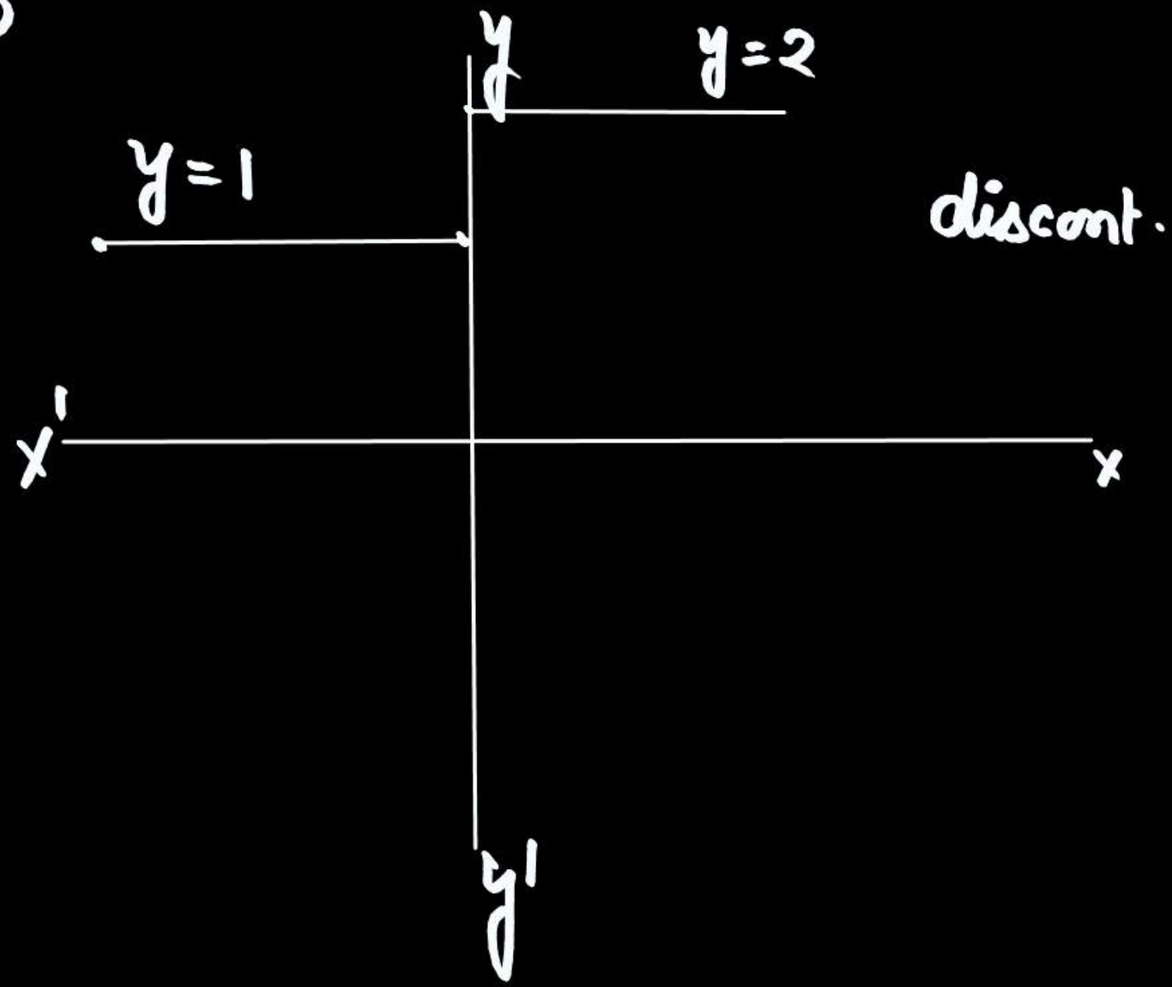
$$F(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$$

L·H·L $\lim_{x \to 1^-} 1 = 1$

R·H·L $\lim_{x \to 1^+} 2 = 2$

$$\boxed{\text{L·H·L} \neq \text{R·H·L}}$$

discont



GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

1. Examine the following functions for continuity at the indicated point :

i) $f(x) = \begin{cases} x^3 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$ at x=0

$\lim_{x \to 0} F(x) = \lim_{x \to 0} x^3 + 3 = 3$

$F(0) = 1$

$\lim_{x \to c} F(x) \neq F(c)$

F(x), donot Conti.

DOWNLOAD GAMMA CAREER CLASSES AAP!

ii) $f(x) = x^3 + 2x^2 - 1$ at $x = 1$

$\lim_{x \to 1} f(x) = 1 + 2 - 1 = 2$

$f(1) = 1 + 2 - 1 = 2$

$\boxed{\lim_{x \to 1} f(x) = f(1)}$

DOWNLOAD GAMMA CAREER CLASSES AAP!

If $f(x) = \frac{x^2-1}{x-1}$ for $x \neq 1$ and $f(x) = 2$ when $x = 1$, show that the functions is continuous at x=1

$$F(x) = \frac{x^2-1}{x-1} \quad x \neq 1$$

$$2 \qquad x = 1$$

$$\lim_{x\to 1} F(x) = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1} (x+1) = 1+1 = 2$$

$$\boxed{F(1) = 2}$$

Hence $\lim_{x\to 1} F(x) = F(1) = 2 \Rightarrow F(x)$ is Continuous.

DOWNLOAD GAMMA CAREER CLASSES AAP!

A functions f is defined as $f(x)=\begin{cases}\frac{x^2-x-6}{x-3}, & x\neq 3\\ 5, & x=3\end{cases}$ show that f is continuous at x=3

$$\lim_{x\to 3} F(x) = \lim_{x\to 3} \frac{x^2-x-6}{x-3} = \lim_{x\to 3} \frac{x^2-(3-2)x-6}{x-3}$$

$$\lim_{x\to 3} \frac{x^2-3x+2x-6}{x-3} = \lim_{x\to 3} \frac{x(x-3)+2(x-3)}{x-3}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+2)}{x-3} = 3+2=5$$

$$\boxed{F(3)=5}$$

Hence $\lim_{x\to 3} F(x)=F(3)$ So function is Continuous.

DOWNLOAD GAMMA CAREER CLASSES AAP!

Is the function f defined by $f(x)=\begin{cases} x, & if\ x \le 1 \\ 5, & if\ x > 1 \end{cases}$ continuous at

i) $x = 0$ ii) $x = 1$ iii) $x = 2$

(i) $\lim_{x \to 0} f(x) = \lim_{x \to 0} x = 0$

$f(0) = 0$

$\lim_{x \to 0} f(x) = f(0)$ So Function is Cont.

L·H·L ≠ R·H·S

(ii) $x = 1$

L·H·L $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x = 1$

R·H·L $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 5 = 5$

Function is not Cont.

DOWNLOAD GAMMA CAREER CLASSES AAP!

$x = 2$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} 5 = 5$

$f(2) = 5$

$L.H.L = R.H.L = f(2)$

$f(x)$ is Cont.

GAMMA CAREER CLASSES

DOWNLOAD GAMMA CAREER CLASSES AAP!

If the function f defined by $f(x)=\begin{cases} 3x+5, if\ x \geq 2 \\ x^2, if\ x < 2 \end{cases}$ continuous at x=2?

L·H·L

$f(x) = x^2$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = \lim_{x \to 2} (2)^2 = 4$$

R·H·L

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x+5) = (3 \times 2 + 5) = 11$$

L·H·L ≠ R·H·L Hence f(x) is not Cont.

DOWNLOAD GAMMA CAREER CLASSES AAP!

Is the function f defined by $f(x) = \begin{cases} \frac{2x^2-3x-2}{x-2}, & if\ x \neq 2 \\ 5, & if\ x = 2 \end{cases}$, continuous at $x = 2$?

$$\lim_{x\to 2} \frac{2x^2-3x-2}{x-2}$$

$$\lim_{x\to 2} \frac{2x^2-3x-2}{x-2} = \lim_{x\to 2} \frac{2x^2-(4-1)x-2}{x-2}$$

$$\lim_{x\to 2} \frac{2x^2-4x+x-2}{x-2}$$

$$\lim_{x\to 2} \frac{2x(x-2)+1(x-2)}{x-2}$$

$$\lim_{x\to 2} \frac{(2x+1)(x-2)}{(x-2)}$$

$$\lim_{x\to 2} (2x+1)$$

$= 2\times 2+1 = 4+1 = 5$

$F(2) = 5$

$\lim_{x\to 2} f(x) = f(2)$

So f(x) Conti.

DOWNLOAD GAMMA CAREER CLASSES AAP!

If $f(x)=\begin{cases} 3x-8, & x \leq 5 \\ 2k, & x>5 \end{cases}$ find k so that f may be continuous at x=5

L·H·L $f(x) = 3x-8$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} (3x-8) = 3\times 5 - 8 = 15-8 = 7$$

R·H·L

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5} 2k = 2k$$

$f(5) = 3(5)-8 = 15-8 = 7$

L·H·L = R·H·L = F(5)

$7 = 2k = 7$

$2k = 7$

$$k = \frac{7}{2}$$

DOWNLOAD GAMMA CAREER CLASSES AAP!